Caltech

MACHINE LEARNING & DATA MINING CS/CNS/EE 155

Deep Learning Part I



can fit binary labels $Y \in \{0, 1\}$

 $\mathbf{w}^{\mathsf{T}}\mathbf{x} + w_0 = 0$ defines the boundary between the classes in higher dimensions, this is a hyperplane



additional features in X result in additional weights in ${f w}$



$$P(Y = 1|X) = \frac{1}{1 + e^{-(\mathbf{w}^{\intercal}\mathbf{x} + w_0)}}$$

use binary cross-entropy loss function

$$\mathcal{L} = -\sum_{i=1}^{N} \left[y^{(i)} \log(P(y_i = 1 | \mathbf{x}^{(i)})) + (1 - y^{(i)}) \log(1 - P(y_i = 1 | \mathbf{x}^{(i)})) \right]$$

$$\text{when } y_i = 1 \text{ make the} \qquad \text{when } y_i = 0 \text{ make the} \text{ output close to } 1 \qquad \text{when } y_i = 0 \text{ make the} \text{ output close to } 0$$

if $Y \in \{-1, 1\}$

$$P(y|\mathbf{x}) = \frac{1}{1 + e^{-y(\mathbf{w}^T\mathbf{x} + w_0)}}$$

use logistic loss function

$$\mathcal{L} = \sum_{i=1}^{N} \log(1 + e^{-y^{(i)}(\mathbf{w}^T \mathbf{x}^{(i)} + w_0)})$$

make sure $\mathbf{w}^T \mathbf{x}^{(i)} + w_0$ and $y^{(i)}$ have the same sign, and $\mathbf{w}^T \mathbf{x}^{(i)} + w_0$ is large in magnitude

how do we extend logistic regression to handle multiple classes? $y \in \{1, \dots, K\}$

approach I

split the points into groups of one vs. rest, train model on each split



approach II

train one model on all data classes simultaneously

$$P(Y = k | X) = \frac{e^{(\mathbf{w}_{k}^{\mathsf{T}} \mathbf{x} + w_{0,k})}}{\sum_{k'=1}^{K} e^{(\mathbf{w}_{k'}^{\mathsf{T}} \mathbf{x} + w_{0,k'})}}$$

softmax function



multi-class logistic regression
$$P(Y = k | X) = \frac{e^{(\mathbf{w}_k^\mathsf{T} \mathbf{x} + w_{0,k})}}{\sum_{k'=1}^K e^{(\mathbf{w}_{k'}^\mathsf{T} \mathbf{x} + w_{0,k'})}}$$

assume probabilities of the form
$$P(Y = k) = \frac{1}{Z}e^{(\mathbf{w}_k^\intercal \mathbf{x} + w_{0,k})}$$

Z is the 'partition function,' which normalizes the probabilities

probabilities must sum to one

$$\sum_{k=1}^{K} P(Y=k) = \sum_{k=1}^{K} \frac{1}{Z} e^{(\mathbf{w}_{k}^{\mathsf{T}}\mathbf{x}+w_{0,k})} = 1$$

therefore
$$Z = \sum_{k=1}^{K} e^{(\mathbf{w}_k^\mathsf{T} \mathbf{x} + w_{0,k})}$$

logistic regression is a linear classifier

linear scoring function is passed through the non-linear logistic function to give a probability output

often works well for simple data distributions

breaks down when confronted with data distributions that are not linearly separable



to tackle non-linear data distributions with a linear approach, we need to turn it into a linear problem

use a set of non-linear features in which the data are linearly separable

one approach:

use a set of pre-defined non-linear transformations

 $X_1, X_2 \rightarrow X_1, X_2, X_1X_2$

linear decision boundary

hyperbolic decision boundary



XOR

to tackle non-linear data distributions with a linear approach, we need to turn it into a linear problem

use a set of non-linear features in which the data are linearly separable

another approach:

logistic regression outputs a non-linear transformation, use multiple stages of logistic regression

$$X_1, X_2 \to X_1 \land X_2, X_1 \lor X_2$$

linear decision boundary

multiple linear decision boundaries

XOR:
$$\neg(X_1 \land X_2) \land (X_1 \lor X_2)$$



XOR

we used multiple stages of linear classifiers to create a a non-linear classifier

as the number of stages increases, so too does the expressive power of the resulting non-linear classifier

depth: the number of stages of processing

the point of <u>deep</u> learning

with enough stages of linear/non-linear operations (depth), we can learn a good set of non-linear features to linearize any non-linear problem

basic operation: logistic regression



multiple operations



layer of artificial neurons

multiple stages of operations



artificial neural network

















 $X_{1,0}^0 = 1$

vectorized form (dot product)



$S^1 = W^1 X^0$

fully *vectorized* form (matrix multiplication)



element-wise non-linearity



and proceed





note: appending biases to W^1 and bias units to X^0 changes

$$N^0 \to N^0 + 1$$





it's just a (deep, non-linear) function!

note: bias appending omitted for clarity



train with gradient descent X^{L} is the output of the network

use $\mathcal{L}(X^L,Y)$ to compare X^L and Y

take gradients $\nabla_{W^{\ell}} \mathcal{L}$ of loss w.r.t. weights at each level ℓ

update the weights to decrease loss, bring X^L closer to Y

$$W^{\ell} \leftarrow W^{\ell} - \alpha \nabla_{W^{\ell}} \mathcal{L}$$



how do we get the gradients? backpropagation

I. Define a loss function.

2. Use *chain rule* to recursively take derivatives at each level <u>backward</u> through the network.

what is the best form in which to present the labels?

what is the best form in which to present the labels?

categorical labels:
$$Y \in \{1, \dots, K\}$$

(multi-class classification)

represent with a single output unit, regress to the correct class?



No, in general, classes do not have a numerical relation

class K is not 'greater' than class K-1

what is the best form in which to present the labels?

categorical labels: $Y \in \{1, \dots, K\}$

(multi-class classification)

represent with multiple output units, regress to an encoding of the correct class (e.g. binary)



No, correct output requires coordinated effort from units

implies arbitrary similarities induced by the coding

what is the best form in which to present the labels?

```
categorical labels: Y \in \{1, \dots, K\}
```

(multi-class classification)

represent with K output units, multi-class logistic regression at the last layer



Yes

captures independence assumptions in the class structure, and is a valid output
output

<u>categorical labels</u>: $Y \in \{1, \ldots, K\}$

(multi-class classification)

represent with K output units, multi-class logistic regression at the last layer

one-hot vector encoding

example K=5





 $Y = 4 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0$

loss function

probability of ${f x}$ belonging to class k

$$P(Y = k) = \frac{e^{\mathbf{w}_k^{\mathsf{T}} \mathbf{x}}}{\sum_{k'=1}^{K} e^{\mathbf{w}_{k'}^{\mathsf{T}} \mathbf{x}}}$$

softmax loss function (cross-entropy)

$$\mathcal{L}_i = -\log P(Y = y_i) = -\log \frac{e^{\mathbf{w}_{y_i}^{\mathsf{T}} \mathbf{x}}}{\sum_{k'=1}^{K} e^{\mathbf{w}_{k'}^{\mathsf{T}} \mathbf{x}}}$$

minimize the negative log probability of the correct class

$$\mathcal{L} = -\sum_{i=1}^{n} \left[y^{(i)} \log(P(y_i = 1 | \mathbf{x}^{(i)})) + (1 - y^{(i)}) \log(1 - P(y_i = 1 | \mathbf{x}^{(i)})) \right]$$

softmax loss is the extension of binary case to multi-class

deep networks are just composed functions

$$X^{L} = \sigma(W^{L}\sigma(W^{L-1}\sigma(\dots\sigma(W^{1}X^{0})))))$$

the loss $\mathcal{L}(X^L, Y)$ is a function of X^L , which is a function of each layer's weights W^ℓ

therefore, we can find $\nabla_{W^{\ell}}\mathcal{L}$ using chain rule



at the output layer

$$\frac{\partial \mathcal{L}}{\partial W^L} = \frac{\partial \mathcal{L}}{\partial X^L} \frac{\partial X^L}{\partial S^L} \frac{\partial S^L}{\partial W^L}$$



at the output layer





at the layer before



general overview - dynamic programming

compute gradient of loss w.r.t. W^L store $\frac{\partial \mathcal{L}}{\partial S^L}$ for $\ell = L - 1, \dots, 1$ use $\frac{\partial \mathcal{L}}{\partial S^{\ell+1}}$ to compute gradient of loss w.r.t. W^ℓ store $\frac{\partial \mathcal{L}}{\partial S^\ell}$

recall $X^{\ell} = \sigma(S^{\ell})$ $S^{\ell} = W^{\ell} X^{\ell-1}$

∂X^ℓ	derivative of non-linearity w.r.t. input
$\overline{\partial S^\ell}$	depends on non-linearity used

$$\frac{\partial S^\ell}{\partial X^{\ell-1}} = W^\ell$$

$$\frac{\partial S^{\ell}}{\partial W^{\ell}} = (X^{\ell-1})^{\mathsf{T}}$$

vanishing gradients



to solve this issue, use non-saturating non-linearities

 $ReLU(S) = \max(0, S)$

<u>pro</u>: keep gradient signal strong at early layers

<u>con</u>: partially linearizes the network, making it less expressive



rectified linear units



adversarial examples



x "panda" 57.7% confidence



"nematode" 8.2% confidence



 $x + \epsilon sign(\nabla_x J(\theta, x, y))$ "gibbon" 99.3 % confidence



Goodfellow et al., 2014 Szegedy et al., 2013



we often want to punish model complexity

deep networks are a powerful model class, making it easier for them to overfit

as in other ML methods, we can regularize by putting a <u>penalty on the weights</u>, and adding this term to the loss function



 $\lambda \sum_{\ell=1}^{L} ||W^{\ell}||_2^2$ L2 regularization

'weight decay'

this can be achieved through L1 or L2 regularization on network's weights

dropout

often, deep networks will learn *entangled* representations, in which the internal representation depends heavily on coordinated activity from multiple units.

this is referred to as 'fragile co-adaptation,' and often generalizes poorly



to encourage units to learn statistically independent features, randomly *dropout* a fraction of the units during each training iteration

something like an 'internal ensemble' of an exponential number of different models during test time, keep all units active

dropout



dropout

visualization of features learned on MNIST digits without dropout with dropout p = 0.5

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redundant features

Srivastava, 2014

early stopping

stop training the model when the validation loss or error plateaus (stops decreasing)

prevents the model from overfitting to the training set



optimization

gradient descent: feed in the entire dataset, calculate gradients for each weight, update the weights, repeat until convergence

with a large model and large dataset, this will be an incredibly slow process

gradient contributions from each data example are averaged

accurate gradient, but one epoch results in a single 'step'



optimization landscape in weight space

optimization

(mini-batch) stochastic gradient descent (SGD): feed in the dataset one batch at a time, calculate gradients for each weight from that batch, update the weights, repeat until convergence, randomly shuffling after each epoch

each batch provides a noisy estimate of the gradient

with an adequate batch size, this is often good enough to head in the right direction

noise in the gradient can actually help prevent getting stuck in local optima



optimization landscape in weight space

optimization

training a deep neural network involves non-convex optimization

convex optimization: with proper learning rate, guaranteed to converge to global optimum

non-convex optimization: no guarantees, may converge to a local optimum

should we be worried about ending up in local optima?

no, not really.

as the number of weights grows, it tends to become easier to escape these local minima. they appear to be mostly saddle points, and most local minima are actually pretty good.



optimization landscape in weight space

what makes a deep network non-convex?

it's the non-linearities!

$$X^{\ell} = \sigma \left(W^{\ell} \sigma \left(W^{\ell-1} \sigma \left(W^{\ell-2} \sigma \left(\cdots X^{0} \right) \right) \right) \right)$$

if we remove the non-linearities...

$$X^{\ell} = W^{\ell} \quad W^{\ell-1} \quad W^{\ell-2} \quad \bullet \quad X^{0}$$
$$\longrightarrow \quad X^{\ell} = W^{1...\ell} \quad X^{0}$$

the network collapses down into a (convex) linear optimization problem

optimization techniques

vanilla stochastic gradient descent

$$W^{\ell} \leftarrow W^{\ell} - \alpha \nabla_{W^{\ell}} \mathcal{L}$$

stochastic gradient descent with momentum momentum momentum $\in (0,1]$ $\mu \leftarrow \beta \mu + \alpha \nabla_W \ell \mathcal{L}$

 $W^\ell \leftarrow W^\ell - \mu$

vanilla SGD

gradient is influence by previous gradient updates

speeds up convergence immensely, prevents optimization from bouncing back and forth too much

rule of thumb: momentum = 0.9



with momentum



optimization techniques

problem: how do we set and anneal the learning rate? why have the same learning rate for all parameters?

adaptive learning rate techniques

<u>adagrad</u>: shrink each parameter's learning rate according to magnitude of sum of past gradients <u>adadelta</u>: similar to adagrad, but with exponentially decaying influence from past gradients <u>RMSprop</u>: similar idea to adadelta

<u>adam</u>: similar to adadelta, but with additional decaying influence from previous gradients (like momentum)





1957 perceptron learning algorithm



- **1980** neocognitron
- **1986** backprop becomes popular
- **1989** convolutional neural networks

neural net winter ll

- 2006 unsupervised pre-training of deep networks
- **2009** use of GPUs for training deep networks
- **2011** unsupervised learning of cat from YouTube
- 2011 deep networks become state-of-the-art for speech recognition
- 2012 deep networks become state-of-the-art for object recognition
- 2012- deep learning boom: ResNets, Neural Turing Machine, deep generative models, deep reinforcement learning, etc.



'Mark I Perceptron at the Cornell Aeronautical Laboratory', hardware implementation of the first Perceptron (Source: Wikipedia / Cornell Library)

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Fukushima, 1980

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Learning Internal Representations by Error Propagation

D. E. RUMELHART, G. E. HINTON, and R. J. WILLIAMS

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Teh et al., 2006

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NVIDIA

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cat filter learned from unsupervised learning, https://googleblog.blogspot.com/2012/06/using-large-scale-brain-simulations-for.html

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Deep Neural Networks for Acoustic Modeling in Speech Recognition

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Geoffrey Hinton, Li Deng, Dong Yu, George Dahl, Abdel-rahman Mohamed, Navdeep Jaitly, Andrew Senior, Vincent Vanhoucke, Patrick Nguyen, Tara Sainath, and Brian Kingsbury

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Krizhevsky, et al., 2012

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1957 perceptron learning algorithm Externa Input itput Controller neural net winter **Read Heads** 1980 neocognitron backprop becomes popular Policy activicity 1986 Value notiron convolutional neural networks 1989 SL DOLOG "OTWOR neural net winter II unsupervised pre-training of deep networks 2006 use of GPUs for training deep networks 2009 2011 unsupervised learning of cat from YouTube He et al., 2015 2011 deep networks become state-of-the-art for speech recognition Graves et al., 2014 Salimans et al., 2016 deep networks become state-of-the-art for object recognition 2012 Minh et al., 2015 deep learning boom: ResNets, Neural Turing Machine, 2012-Silver et al., 2016 deep generative models, deep reinforcement learning, etc.

recent neural network boom

deep learning is <u>hot</u> right now

what started this boom?



68

recent neural network boom

who started this boom?



geoff hinton

'the big three'

...and others

yoshua bengio





yann lecun

recent neural network boom

who started this boom?

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geoff hinton

yoshua bengio



http://www.andreykurenkov.com/writing/a-brief-history-of-neural-nets-and-deep-learning-part-4/

yann lecun



neural network winters

research in neural networks has followed a boom-bust cycle



neural network winters

we're obviously in a boom period. is this time any different?

yes and no

predicting the future of A.I. research progress is notoriously difficult

it's possible that deep learning research will hit a wall, where either it becomes too computationally expensive for most individuals to do groundbreaking research or a new, better approach comes along

however, deep learning is now commercially successful. most large tech companies profit from it. so as long as it remains the state-of-the-art approach, there will be (funding for) basic research.







facebook
beware the hype

just because deep learning is booming, it doesn't mean human-level A.I. is 'just around the corner'

saying that something is 5-10 years away is almost always just pure speculation

if someone tries to tell you that deep learning works off of the same principles as the brain, tell them that they don't know what they're talking about

cool graphics, but highly inaccurate





does deep learning live up to the hype?

deep learning is nothing more than a passing phase

non-believers

deep learning is the dawn of general A. I.

believers

we don't have a great intuition for how or why it works

models are often uninterpretable a.k.a 'black box'

doesn't actually work like the brain

consistently beats all other methods on vision, NLP benchmarks

learn your features instead of hand-coding them

it's biologically inspired!

where does deep learning fit in?

deep learning is a useful method for approximating complicated, hierarchical functions. this makes it well-suited for many A. I. tasks

but ultimately it's just a tool for linearizing non-linear data. it doesn't *replace* other machine learning techniques, rather, it *enhances* them

> still much work to be done in understanding these models -what do they learn? -how do we train them more efficiently? -architectural principles?

better methods will be developed eventually, but they will almost certainly involve -hierarchies -learned features -many parameters when to use deep learning?

try a simple method first

deep learning requires *compute* and *data* unless you have both, deep learning won't work

deep learning works by hierarchically sectioning non-linear surfaces i.e. deep learning works best on non-linear data with hierarchical structure

why not other methods?

deep learning's power is in its depth

most other methods are not capable of depth or if they are, they are difficult to train

next lecture

convolutional neural networks

